Nonreciprocity of Phase Constants, Characteristic Impedances, and Conductor Losses in Planar Transmission Lines with Layered Anisotropic Media

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Abstract— **The nonreciprocal characteristics of the planar transmission lines with the layered structures, including the magnetized ferrite, are analyzed based on the versatile hybrid-mode formulation that is applicable to various types of shielded and open structures. Accurate and efficient numerical procedure presents the frequency-dependent nonreciprocal values of the phase constants, characteristic impedances, and attenuation constants, taking the finite metallization thickness into consideration. Numerical results show that the nonreciprocal and metallization thickness effects are dominant parameters on the propagation characteristics, and these effects are different, depending on the types of the planar transmission lines.**

I. Introduction

A LARGE NUMBER of papers have been devoted to planar transmission lines with anisotropic medium. An anisotropic medium of practical importance is magnetized ferrite, which is often used to realize nonreciprocal devices in microwave and millimeter-wave integrated circuits [1], [2]. Highly accurate hybrid-mode analysis is indispensable to the nonreciprocal circuit design. The spectral domain approach (SDA) [3] has been applied successfully to the analysis of the planar transmission lines with magnetized ferrite [4]. The formulation procedure of SDA was extended by the author to analyze the multilayered structures, including the spacers or overlays in addition to the magnetized ferrite, and to treat the coupled and asymmetrical structures [5], [6]. Also, the procedure takes the metallization thickness effect into consideration, and it reveals the effect of the finite thickness of metallization on the nonreciprocal propagation [5], [6]. The effect becomes more serious in the higher frequency range, where the cross-sectional dimensions are so small and the relative thickness of the metallization becomes significant [5]. However, there exist only the metallization thickness effects on the phase constants of the planar transmission lines with magnetized ferrite. There is a lack of the analytical method that can afford the nonreciprocal characteristics of the characteristic impedances and the attenuation due to the imperfect conductor.

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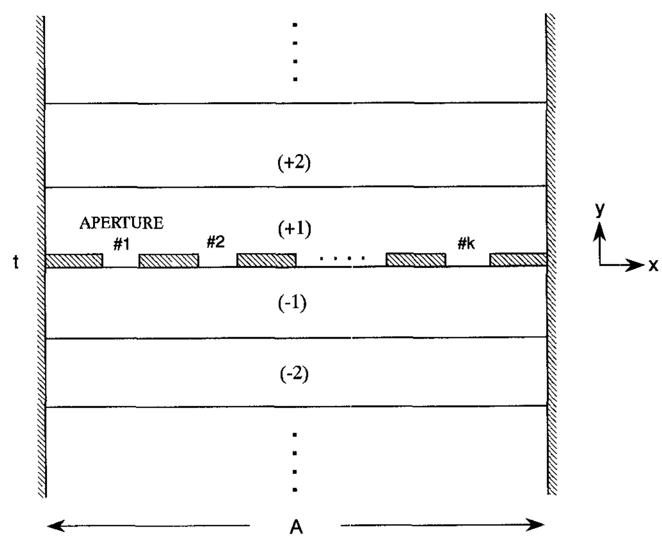

Fig. 1. General structure of planar transmission lines with layered anisotropic media.

The formulation procedure described in the present paper is based on the extended spectral domain approach (ESDA) and is extended further to evaluate the characteristic impedances and the conductor losses in addition to the phase constants for the various planar transmission lines including the magnetized ferrite with additional dielectric layers. Due to the hybrid-mode propagation, the definition for the frequency-dependent characteristic impedance is not uniquely specified. Suitable definitions are selected for each case, and the accurate evaluations are performed based on the definitions. It is well known that the calculation of the conductor loss, neglecting the metallization thickness, is principally erroneous. The introduction of finite metallization thickness would overcome the computational difficulty of the conductor losses, and it is applicable to the loss calculations of the thicker as well as thinner line conductors, where the fields penetrating from both surfaces of the conductor overlap each other and attenuation becomes more significant.

II. Theory

Fig. 1 shows the cross-section of the general structure of the planar transmission lines, which consist of printed con-

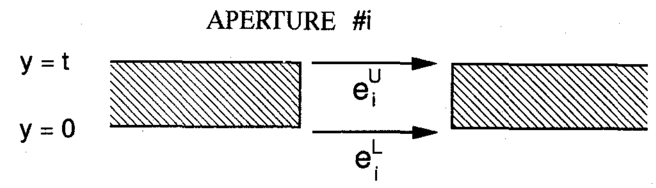

Fig. 2. Source fields of i-th aperture.

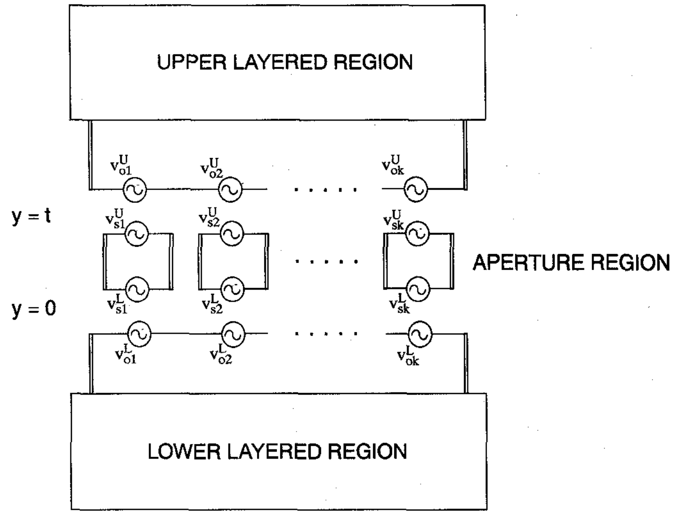

Fig. 3. Equivalent circuit in the y-direction.

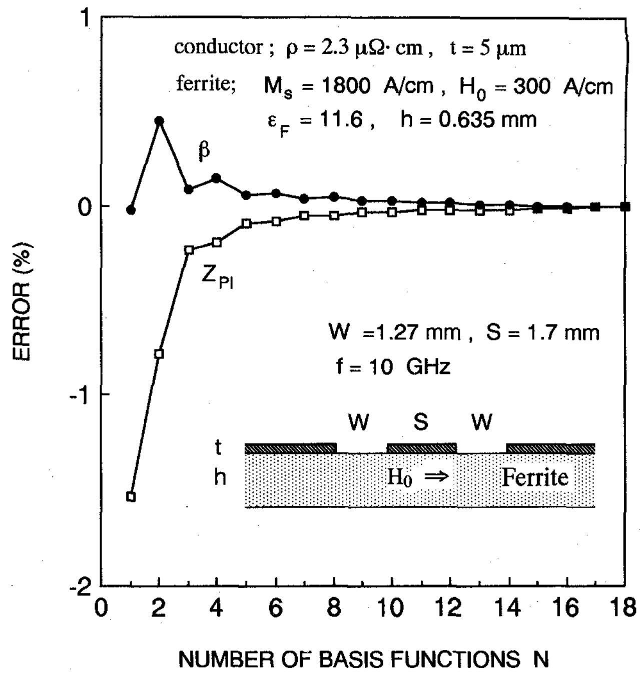

Fig. 4. Convergenceof calculated results of β and Z_{PI}.

ductors of arbitrary thickness t with layered isotropic and/or anisotropic media. The figure shows the shielded structures with the side walls, e.g., fin lines and shielded strip lines, but the approach described in the following is quite versatile and is applicable to the open structures without the side walls, e.g., slot lines and CPW's. Numerical results both for shielded and open structures will be presented. When the layer (i) is the ferrite and is magnetized in the x direction, the permeability tensor of the layer is expressed as [4]–[6]

$$\overline{\overline{\mu}}^{(i)} = \mu_0 \begin{bmatrix} 1 & 0 & 0 \\ 0 & \mu_r & j\kappa \\ 0 & -j\kappa & \mu_r \end{bmatrix}, \tag{1}$$

where μ_r and κ are dependent on the operating frequency ω, the applied dc magnetic field H_0, and magnetization of the ferrite $4\pi M_s$,

$$\mu_r = 1 - \frac{\gamma^2 H_0 4\pi M_s}{\omega^2 - (\gamma H_0)^2}, \quad \kappa = \frac{\gamma 4\pi M_s \omega}{\omega^2 - (\gamma H_0)^2}, \tag{2}$$

where γ is the gyromagnetic ratio.

Following the formulation procedure of the extended spectral domain approach (ESDA) [5], [6], electromagnetic fields in each region are Fourier transformed with respect to the x-direction. It should be noted that the open structures without the side walls cannot be simulated by the shielded structures even if the separation A between the side walls is chosen very large. That is, the higher-order modes of the open structures are similar to the surface modes [7], whereas those of the shielded structures are the waveguide modes [8]. Therefore, the Fourier integral representation is used for the open structures, whereas the Fourier series is used to satisfy the boundary conditions on the side walls of the shielded structures. But, the formulation procedures after the transformation are common to both structures. The fields in the aperture region ($t > y > 0$) are represented by the Fourier series. The continuity conditions at the interfaces of layered structures are also Fourier transformed. Then, the aperture fields are introduced. The fields in the i-th aperture are designated as $\mathbf{e}_i^U$ (at $y = t$) and $\mathbf{e}_i^L$(at $y = 0$) (Fig. 2), and their Fourier transforms are v_{oi}^U and v_{oi}^L for the upper and lower layered regions, respectively. The Fourier transforms for the aperture region are v_{si}^U and v_{si}^L. Therefore, in the spectral region, the equivalent circuits in the y-directions can be derived as Fig. 3. The transformed fields in each region can be related to the transformed aperture fields $v_{oi}^U, v_{oi}^L, v_{si}^U$ and v_{si}^L. The transformed electromagnetic fields in the subregions are transformed inversely and the fields can be expressed in terms of the aperture fields $\mathbf{e}_i^{U(x)}$ and $\mathbf{e}_i^L(x)$ as;

$$\begin{aligned} E^{(m)}(x,y,z) = \sum_i \int_{x_i'} \Big\{ & \overline{\overline{T}}_{Ui}^{(m)}(x,y \mid x')e_i^U(x') \\ & + \overline{\overline{T}}_{Li}^{(m)}(x,y \mid x')e_i^L(x') \Big\} dx' e^{-j\beta z} \\ E^{(m)}(x,y,z) = \sum_i \int_{x_i'} \Big\{ & \overline{\overline{Y}}_{Ui}^{(m)}(x,y \mid x')e_i^U(x') \\ & + \overline{\overline{Y}}_{Li}^{(m)}(x,y \mid x')e_i^L(x') \Big\} dx' e^{-j\beta z} \end{aligned} \tag{3}$$

where the $\overline{\overline{T}}$'s and $\overline{\overline{Y}}$'s are the dyadic Green's functions.

By enforcing the continuities of the magnetic fields at the aperture surfaces $y = t$ and 0 to (3), we obtain the integral equations for the aperture fields $\mathbf{e}_i^U(x), \mathbf{e}_i^L(x)$ and implicitly the phase constant β. The determinantal equation for the phase constant β is obtained by applying Galerkin's procedure to the integral equations.

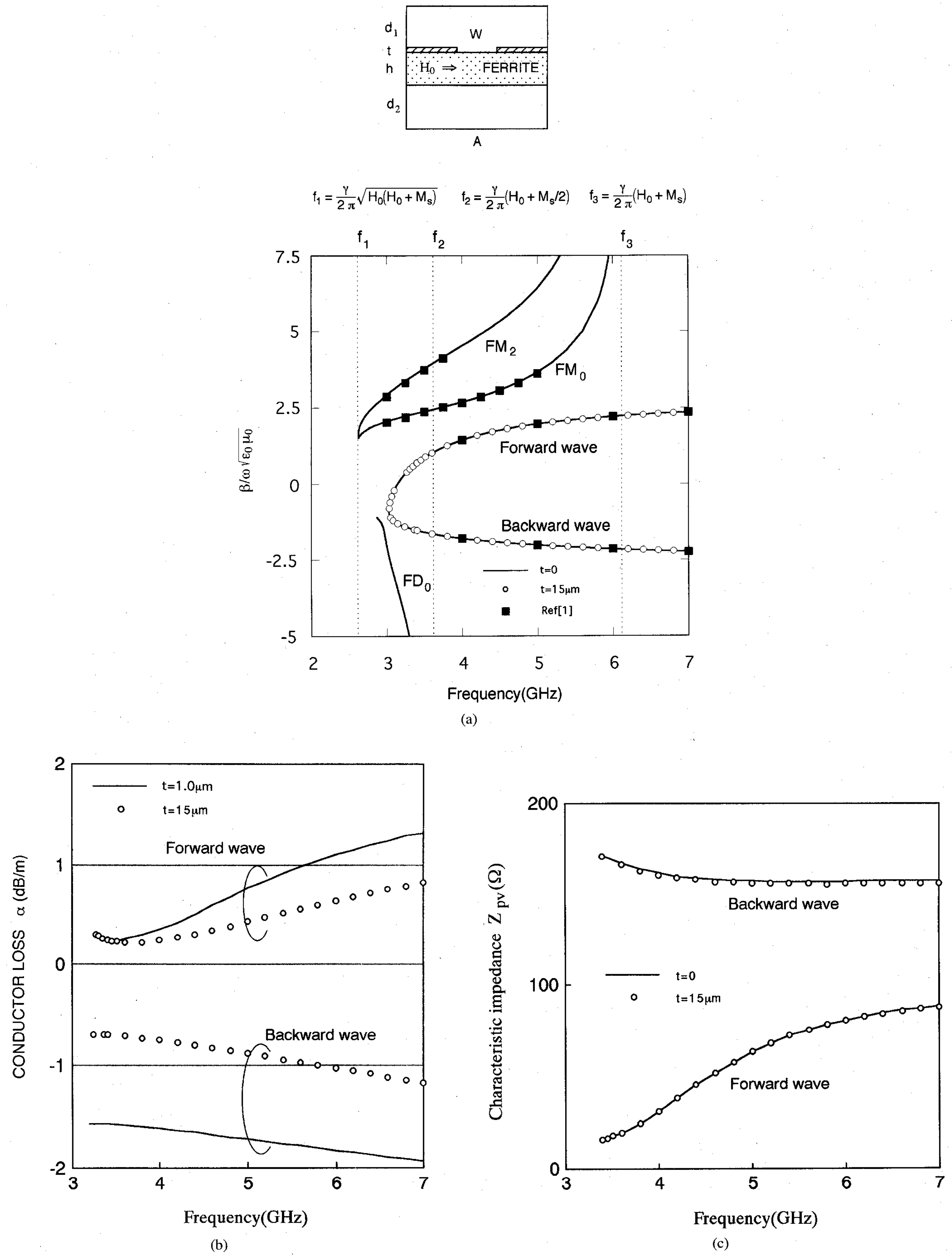

Fig. 5. Frequency-dependentcharacteristics of the fin line with single-layered ferrite.(a) Phase constant,(b) Attennuation constant,(c) Characteristic impedance Z_{pv}.conductor: $\rho = 2.3\ \mu\Omega$·cmferrite: $\varepsilon_F = 13.4$, $h = 2$ mm, $M_s = 1.42$ kA/cm, $H_0 = 0.318$ kA/cm$W = 2$ mm, $A = 20$ mm, $d_1 = 6$ mm, $d_2 = 10$ mm.

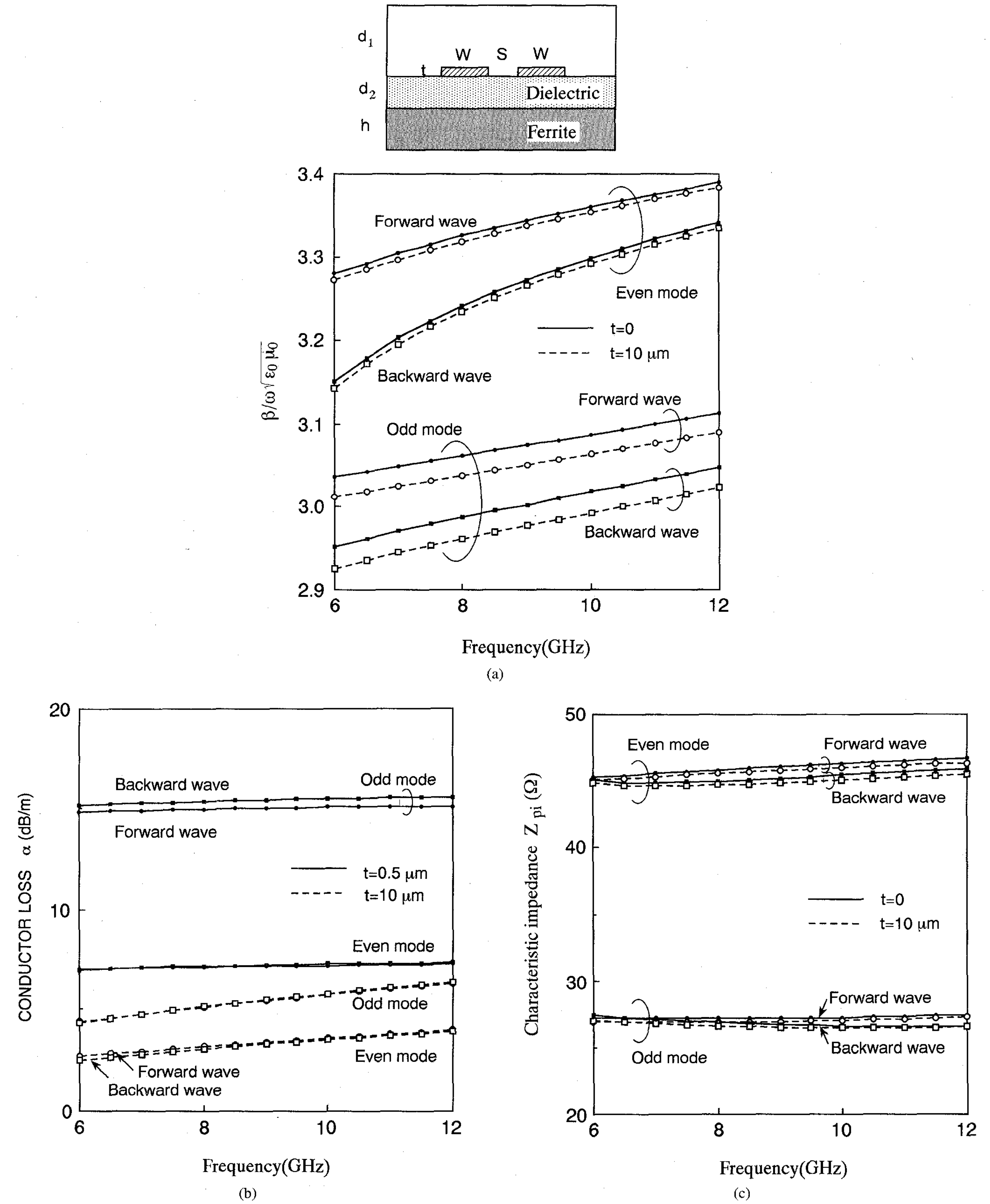

Fig. 6. Frequency-dependent characteristics of the dielectric-ferrite (DF) double-layered coupled strip line.(a) Phase connstant. (b) Attenuation constant. (c) Characteristic impedance $Z_{\rm pi}$conductor: $\rho = 2.3\ \mu\Omega \cdot$ cm dielectrics: $\varepsilon_D = 20$, $d_2 = 0.2$ mm, ferrite: $\varepsilon_F = 11.6$, $h = 0.635$ mm, $M_s = 1800$ A/cm, $H_0 = 300$ A/cm, $S = 0.4$ mm, $W = 1$ mm, $A = 10$ mm, $d_1 = 10$ mm.

The phase constant β is, in turn, substituted into (3) to obtain the electromagnetic fields in each region. The voltage, current, and average power flow along the z direction are calculated by using these fields. Then, characteristic impedances are evaluated based on the appropriate definitions [8]. The proper choice of the impedances for each structures is discussed later.

The procedure explained above neglects the losses. The evaluation of the conductor losses is explained in the following. A rigorous loss calculation is available [9], but it requires enormous computational labor even for simpler structures and it is not easy to apply to the cases with magnetized ferrites. Instead, attenuation due to imperfect conductors is accounted for by the perturbational procedure [10]. But, the conventional perturbational scheme, i.e., the power lost in the conductors P_C is evaluated by the surface integral of the tangential component of the magnetic field $\mathbf{H}_t$ over the conductor surface C, is avoided here. The conventional scheme is based on the assumption that conductor thickness t is sufficiently greater than the skin depth $\delta (t > 3\delta)$, therefore it cannot be applied to the thinner conductors, where the fields penetrating from both surfaces of the conductor overlap each other and attenuation become more significant. ESDA, based on the finite-metallization thickness model, can be combined with the perturbation method to evaluate the loss characteristics for the lines with thicker as well as thinner conductors [11]. The power lost in the conductors P_C is evaluated by the integral of the power dissipation $\sigma|\mathbf{E}|^2$ over the coductor region [11]

$$P_C = \frac{1}{2}\int_{S_c} \sigma|\mathbf{E}|^2 dS. \tag{4}$$

III. Numerical Procedure and Results

In the numerical method based on Galerkin's procedure, the unknown aperture fields $\mathbf{e}^U(x), \mathbf{e}^L(x)$ are expanded in terms of the appropriate basis functions:

$$e^U_{\substack{x\\z}}(x') = \sum_{k=1}^{N_x} a_{\substack{xk\\z}} f_{\substack{xk\\z}}(x),$$

$$e^L_{\substack{x\\z}}(x') = \sum_{k=1}^{N_x} b_{\substack{xk\\z}} f_{\substack{xk\\z}}(x). \tag{5}$$

Preliminary computations have been carried out to show the validity of the method. Fig. 4 shows the convergence of the calculated results with respect to the number of basis functions for symmetrical CPW with the magnetized ferrite. Metallization thickness is chosen to be 5 μm, and the following basis functions are used [10], [11]:

$$f_{xk}(x) = \left[1 - \left\{\frac{2(x-c)}{W}\right\}^2\right]^{-\frac{1}{3}} C_{2k}^{1/6}\left\{\frac{2(x-c)}{W}\right\},$$

$$f_{zk}(x) = \left[1 - \left\{\frac{2(x-c)}{W}\right\}^2\right]^{\frac{2}{3}} C_{2k-1}^{7/6}\left\{\frac{2(x-c)}{W}\right\}$$

(c: center of the aperture) (6)

where $C_k^{\mu}(x)$ are Gegenbauer polynomials. f_{xk} and f_{zk} of (6) exhibit the $\delta^{-1/3}$ and $\delta^{2/3}$ variations near the conductor edge, respectively and they represent the edge singularities of the finite metallization thickness case more properly than the conventional basis functions for the zero metallization case [7]. The number of basis functions is increased up to N =18, and the percent error for the N =18 case is shown for the phase constant and characteristic impedance. Because of the hybrid-mode propagation in this structures, the definition for the frequency-dependent characteristic impedance is not uniquely specified. For symmetrical CPW, three definitions are possible [8], i.e., power-voltage basis; $Z_{\rm PV} = V_0^2/P_0$, power-current basis; $Z_{\rm PI} = P_0/I_0^2$, and voltage-current basis; $Z_{\rm VI} = V_0/I_0$, where V_0 and I_0 are the voltage difference between conductors and the total current on the signal conductor, respectively. P_0 is the average power flow along the z direction, and it can be obtained by integrating the z component of Poynting vector over the cross-section. However, for the asymmetrical CPW case considered later, the voltage difference over the right slot is different from that over the left slot except at zero frequency (the quasi-static case), and the only possible definition for the asymmetrical case is the power-current basis $Z_{\rm PI} = P_0/I_0^2$. This definition is used for symmetrical CPW, too, in Fig. 4. Reasonable convergence is observed both for the phase constant and characteristic impedance.

Numerical calculations of the simplest shielded structure of the fin line with single ferrite substrate are performed in Fig. 5. The nonreciprocal phase constants have been evaluated for this structure and the frequency dependence of the phase constant is compared with available data [1]. Good agreement with the published data is obtained over the frequencies. The numerical results by [1] are based on the assumption that the metallization thickness is zero, whereas the present method can afford the values of thick (t = 15 μm) as well as zero metallization ($t = 0$). The effect of the metallization thickness on the phase constants is not significant for this structure, but the introduction of the finite metallization thickness into the analysis makes the loss calculations possible. Fig. 5(b) shows the frequency dependence of the attenuation constants. The nonreciprocal characteristics as well as the metallization thickness effects of the attenuation constants are observed significantly. In the backward ($-z$ direction) propagation case, the electromagnetic fields are more concentrated near the fin conductors in magnetized ferrite, which results in the larger conductor loss and the larger metallization thickness effect on the loss. Fig. 5(c) shows the frequency dependence of the characteristic impedance. For the fin-line configuration, the current cannot be evaluated uniquely, and only the available definition of the characteristic impedance is the power-voltage basis definition $Z_{\rm pv}$.

Planar waveguides with a single-layered ferrite substrate do not exhibit adequate nonreciprocity, and additional layers, such as spacer or overlay, are introduced to increase the nonreciprocity [4]–[6], [12]. The present procedure is quite versatile and applicable easily to multi-layered structures of various types of planar transmission lines, e.g., strip lines, fin-lines, symmetrical and asymmetrical CPW's. Fig. 6 shows the frequency-dependent characteristics of the coupled strip line with composite dielectric-ferrite (DF) substrate. The figures include the nonreciprocal characteristics and the metallization thickness effect. The metallization thickness has greater influence on the dispersion characteristics of the odd mode than those of the even mode (Fig. 6(a)), but the metallization thickness effects are smaller than the nonreciprocal effects,

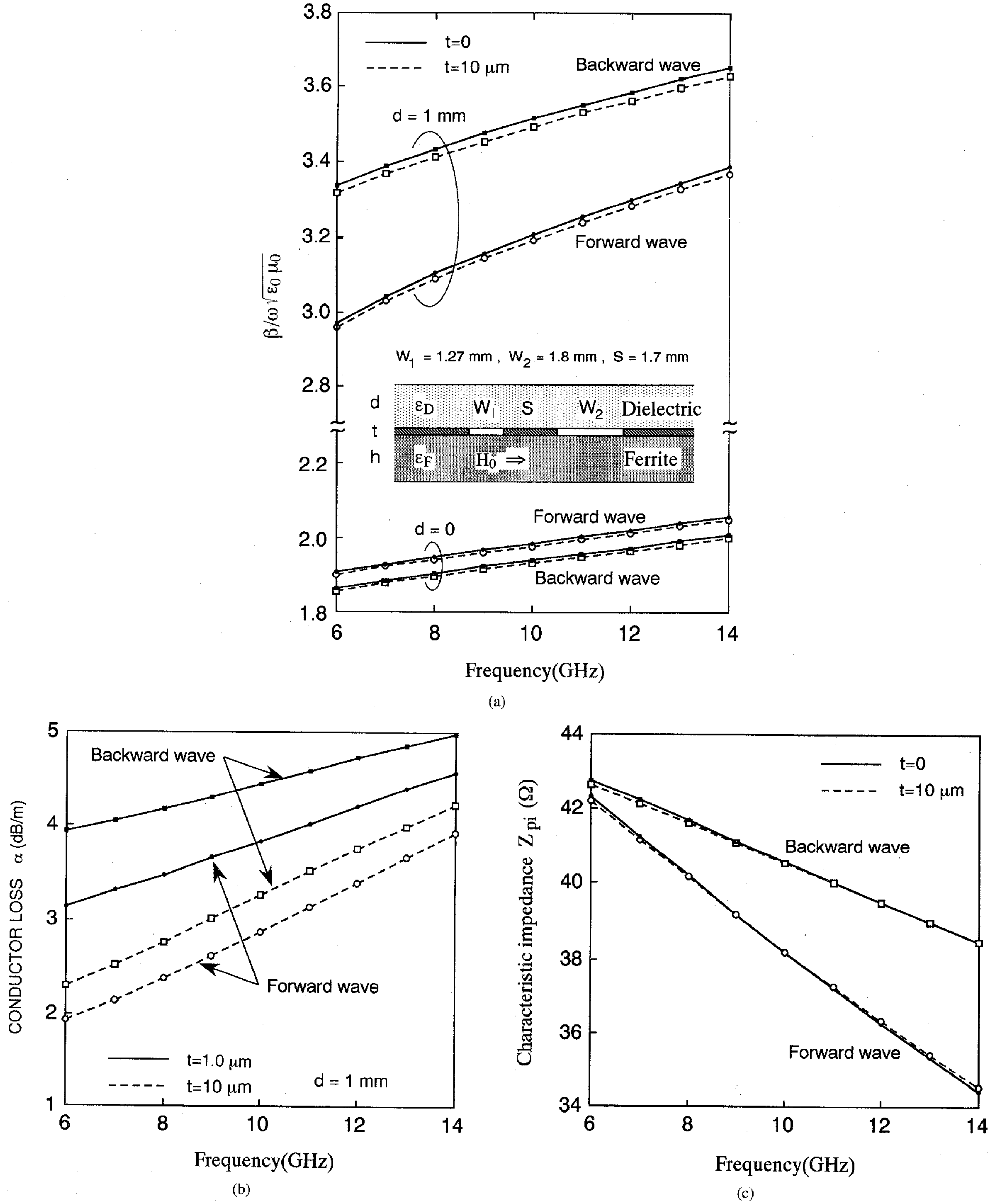

Fig. 7. Frequency-dependentcharacteristics of ACPW with ferrite and dielectric overlay. (a) Phase constant. (b) Attenuation constant. (c) Characteristic impedance Z_{pi}, $\varepsilon_F = 11.6$, $h = 0.635$ mm, $M_s = 1800$ A/cm, $H_0 = 300$ A/cm,$\varepsilon_D = 20$.

the differential phase shift $\Delta\beta = \beta_f - \beta_r$, for this structure. Also, we mention that the frequency-dependences of the field distribution of the odd mode are larger those of the even mode [11], then the frequency-dependences of $\Delta\beta = \beta_f - \beta_r$ is larger for the odd mode case. Fig. 6(b) shows the conductor losses of coupled strip lines. The metallization thickness is a dominant parameter for the conductor losses, and the nonreciprocity of the conductor losses is rather small. It should be noted that the loss value of the odd-mode is larger than that of the even-mode. In the odd mode, the electromagnetic fields are concentrated between the strip conductors, which results in the increase in the current density near the edges and the increase in the conductor loss. The characteristic impedances in Fig. 6(c) is calculated by the power-current definition Z_{PI} for this structure.

Fig. 7 shows the frequency-dependent characteristics of the asymmetrical coplanar waveguide with dielectric overlay (the sandwich asymmetrical CPW). The introduction of the dielectric overlay ($d = 1$ mm) makes the differential phase shift $\Delta\beta = \beta_f - \beta_r$ much larger than the case without the overlay ($d = 0$) (Fig. 7(a)). Also, the nonreciprocity of the conductor losses is observed significantly for this structure (Fig. 7(b)).

IV. Conclusion

A hybrid-mode formulation procedure is describe to evaluate the nonreciprocal characteristics of the various planar transmission lines with the layered structures including the magnetized ferrite. The procedure based on the extended spectral domain approach (ESDA) is developed taking the metallization thickness effect into consideration, and it is applicable to the thicker as well as thinner line conductors without any computational difficulty.

Numerical computations are performed by using the basis functions that represent the actual field variations more properly near the edge of the conductor of finite thickness and they show the good convergence. The nonreciprocal properties as well as the metallization thickness effects on the characteristic impedances and the conductor losses are demonstrated for the first time. The method is quite versatile, and is applicable to various-types of planar transmission lines with the magnetized ferrite. Numerical results show that the nonreciprocal and metallization thickness effects are dominant parameters on the propagation characteristics, and this effects are so different according to the types of the planar transmission lines.

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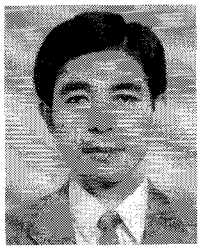

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